Correlation studies in a spin imbalanced fermionic superfluid in presence of a harmonic trap

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We investigate different physical properties of a spin imbalanced fermionic superfluid described by an attractive Hubbard model in presence of a harmonic trap. To characterize the ground state of such a system, we compute various correlation functions, such as pairing, density and current correlations To underscore the effect of the trap, we compute these quantities as a function of strength of the trapping potential. A distinguishing feature of a superfluid state with population imbalance, is the appearance of a phase with spatially modulating gap function, called the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase which is present for both with and without trap. Trapping effects induce lowering of particle density and yields an *empty* system in the limit of large trapping depths.

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I. INTRODUCTION

The experimental realization of Bose-Einstein condensation in dilute alkali gases[1–5] has triggered vast exploration of the field of ultracold, degenerate gas of fermionic atoms in presence of a harmonic trap. Achieving degeneracy in a system of fermionic atoms is a well known hurdle as *s*-wave collisions between fermions in the same state are blocked by the exclusion principle (scattering in higher angular momentum channels, such as *p*-wave, are anyway weak). However researchers continued to access the degeneracy limit and the rare earths are recent additions on the list[6, 7]. In most atomic gas experiments, the experimental challenge of producing ultracold temperatures is achieved by confining the atoms using a trapping potential. In fact, the velocity or momentum distribution of the gas is obtained via a technique called 'time of flight' which involves a *sudden turn off* of the trapping potential and hence wait for an *expansion time*. The trapping potential is produced due to interaction between a gaussian laser beam and the induced atomic dipole moment. At the bottom of the trap, the gaussian profile can pretty much be approximated by a harmonic function, except for very shallow trap depths.

In this paper, we shall incorporate the optical trap depth effects by a harmonic trap. So our system comprises of a population imbalanced superfluid in the presence of a harmonic trap which has a minimum (deepest) at the center of the (two-dimensional) lattice and gradually increases towards the edges[8, 9]. Further we shall focus on the effect of the trap on properties of the ground state by reviewing different correlation functions that characterize the state. These are sure to provide important insights on the nature of the polarized Fermi gas.

A convenient technique to emphasize the effect of the trap is to examine the scenario in absence of a trap and compare it with those for various trap depths. Further, since the presence of a harmonic trap forces the system, which is a two-dimensional superfluid of Fermi atoms, to loose translational invariance, it is prudent to investigate correlations in the real space. In this paper, we include several of them such as, gap parameter, off-diagonal long range order, pair-pair, density-density and current-current correlations to study the ground state of the population imbalanced Fermi gas in presence of a harmonic trap. The above list may not be exhaustive, but are certainly useful to characterize a state that incorporates physics of superfluidity in presence of combined effects of population imbalance and harmonic trap.

The results that we obtain have two special features. A phase with a finite momentum Cooper pairing and characterized by a spatially modulating order parameter, exists for moderate values of polarization of the participating species of fermions. This state, called as the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state, persists for both with and without trap[10]. However large imbalance gives way to a fully polarized Fermi liquid[11]. Another interesting artifact of the presence of the trap occurs in the form of loss in density of the constituent particles of the system. This is inevitable since the chemical potential is required to be kept fixed to attain a population imbalance of the participating species and thus is crucial for observing the FFLO phase. Hence in the limit of large trap depth, we obtain an almost *empty* system. This almost *empty* system has some features that mimic an insulating phase, such as a vanishing superconducting gap, an increasing spectral gap and a dramatic reduction

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in mobility of the particles as a function of the trapping potential. It should be noted that a gradual emergence of such an *empty* system with increasing trapping effects *can not* be termed as a transition to an insulating state which are otherwise present in the repulsive version of the Hubbard model[12]. Such transition have been experimentally realized in case of bosonic atoms[13] that are supported by theoretical studies[14].

The study of population imbalanced gases of fermionic atoms in presence of a harmonic trap has a reasonable history[8, 9, 15–17]. Density distribution of the fermionic atoms in the trap has been a central issue and a phase separation is predicted with the paired atoms confined at the center of the trap, while the unpaired ones find place towards the periphery[18, 19]. However a concomitant tuning of the spin imbalance and trap depth was lacking. This paper fills the void and explores the interesting physics associated with it.

We organize our paper in the following fashion. In order to fix notations, we provide a description of the attractive Hubbard model in presence of a harmonic trap and the standard mean field Bogoliubov de Gennes (BdG) theory which are used here. In the next section, we present results for different correlation functions in the presence of a trap and compare with the results when the trapping effects are switched off. For a range of population imbalance caused by a constant Zeeman field in our model, signature of FFLO phase is observed. The FFLO phase is characterized by weaker correlations but has a robust presence both with and without trap effects[8]. Further as a function of the trap depth, we demonstrate a gradual lowering of the particle density in table I. A probe into the nature of the almost *empty* state yields a zero superconducting gap, an increasing spectral gap and a reduced average kinetic energy of the particles as the strength of confinement is enhanced. These features may indicate emergence of an insulating-like state, however when considered along with a gradual vanishing of particle density, it rules out the possibility of a transition from a superfluid to an insulator. We conclude with a brief discussion of our results.

II. MODEL AND FORMALISM

In crystal lattices, where electrons are the main carriers that contribute to transport properties of materials, obtaining a model Hamiltonian that describes the physical properties satisfactorily is a difficult task owing to a highly complicated band structure. However a gaseous system of fermionic atoms in a confining potential is a much neater realization of the simplest model that incorporates strong correlations between atoms at short distances. An attractive version of the two-dimensional Hubbard model in which the atoms interact attractively via the on-site interaction, U in presence of a constant Zeeman field, h and a harmonic trapping potential at site r_i, V_i is written as,

$$\begin{aligned} \mathcal{H} = & -t \sum_{\langle ij\rangle,\sigma} (c^\dagger_{i\sigma} c_{j\sigma} + \text{H.c.}) \\ & - |U| \sum_i (n_{i\uparrow} - \frac{1}{2})(n_{i\downarrow} - \frac{1}{2}) + \sum_{i,\sigma} (V_i - \mu + \sigma h) n_{i\sigma} \end{aligned} \tag{1}$$

μ denotes the chemical potential, t is the hopping matrix element between the nearest neighbours of a two dimensional square lattice. The creation (annihilation) operator for fermionic atoms corresponding to spin σ is $c^\dagger_{i\sigma}$ ($c_{i\sigma}$). The excess of one species of atoms (say with spin-$\uparrow$) over another is controlled by the magnetic field h (or equivalently an effective chemical potential, $\mu' = \sigma h - \mu$). V_i is assumed to be of the form,

$$V_i = V_0 (r_i - r_0)^2 \tag{2}$$

where V_0 is the strength of the trapping potential and r_0 is the position where the center of the trap lies and is located at the center of the lattice. Thus the potential is minimum (deepest) at the center of the lattice and is maximum (shallow) at the edges. All of U, h, μ and V_0 are expressed in units of hopping strength, t which is typically of the order of an eV. A mean field decoupling of the interaction term in Eq. (1) yields the effective Hamiltonian of the form,

$$\mathcal{H}_{eff} = \sum_{ij,\sigma} \mathcal{H}_{ij\sigma} (c^\dagger_{i\sigma} c_{j\sigma} + \text{H.c.}) + \sum_i \left[\Delta_i c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} - \Delta^*_i c_{i\uparrow} c_{i\downarrow} \right] \tag{3}$$

here $\Delta_i = -|U| \langle c_{i\downarrow} c_{i\uparrow} \rangle$ is the gap parameter for the fermionic superfluid. $\mathcal{H}_{ij\sigma} = -t\delta_{i\pm 1 j} + (V_i - \mu - U\delta n_{i\bar{\sigma}} + \sigma h)\delta_{ij}$ where $\delta n_{i\bar{\sigma}} = n_{i\bar{\sigma}} - 1/2$ with $\langle n_{i\sigma} \rangle = \langle c^\dagger_{i\sigma} c_{i\sigma} \rangle$ and $\bar{\sigma} = -\sigma$.

Eq. (3) is hence diagonalized using Bogoliubov transformation which yields,

$$\begin{pmatrix} \mathcal{H}_{ij\sigma} & \hat{\Delta}_i \\ \hat{\Delta}_i^* & -\mathcal{H}^*_{ij\bar{\sigma}} \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}_i) \\ v_n(\mathbf{r}_i) \end{pmatrix} = E_n \begin{pmatrix} u_n(\mathbf{r}_i) \\ v_n(\mathbf{r}_i) \end{pmatrix} \tag{4}$$

where $u_n(\mathbf{r}_i)$ and $v_n(\mathbf{r}_i)$ are the BdG eigenvectors satisfying $\sum_n [u^2_n(\mathbf{r}_i) + v^2_n(\mathbf{r}_i)] = 1$ for all $\mathbf{r}_i$ and E_n are the eigenvalues.

The gap parameter and density (and hence magnetization, $m_i = \langle n_{i\uparrow} \rangle - \langle n_{i\downarrow} \rangle$, see Eq. (6)) in terms of the eigenvectors $u_n(\mathbf{r}_i)$ and $v_n(\mathbf{r}_i)$ at a temperature T are given by,

$$\begin{aligned} \Delta_i = & - |U| \sum_n [u_n(\mathbf{r}_i) v^*_n(\mathbf{r}_i) f(E_{n\uparrow}) \\ & - u_n(\mathbf{r}_i) v^*_n(\mathbf{r}_i) f(-E_{n\downarrow})] \end{aligned} \tag{5}$$

$$\langle n_{i\sigma} \rangle = \sum_n \left[|u_n(\mathbf{r}_i)|^2 f(E_{n\sigma}) + |v_n(\mathbf{r}_i)|^2 f(-E_{n\bar{\sigma}}) \right] \tag{6}$$

where $f(E_{n\sigma})$ is the Fermi distribution function. Δ_i and $\langle n_{i\sigma} \rangle$ are obtained self-consistently from Eq. (5) and Eq. (6) at each lattice site.

It may be noted that a number of self-consistent solutions may exist for the gap parameter, Δ_i corresponding to one set of parameters with different initial guesses. The winner among these will be decided by computing the free energies, $\mathcal{F}$ computed with respect to the free energy of vacuum (at zero temperature) and is given by[20],

$$\begin{aligned}\mathcal{F} &= \sum_{n\sigma} E_{n\sigma}\left[f(E_{n\sigma}) - \sum_i |v_n(\mathbf{r}_i)|^2\right] \\ &+ |U|\sum_i \langle n_{i\uparrow}\rangle\langle n_{i\downarrow}\rangle + \frac{1}{|U|}\sum_i \Delta_i^2 - \frac{|U|N}{4}\end{aligned} \quad (7)$$

We further compute higher order correlations which are likely to be more illustrative in characterizing the phase. We begin with off-diagonal long range order (ODLRO) which characterizes the superfluid phase and can be useful in bringing out distinction between a zero net center of mass momentum pairing (BCS) and a finite momentum pairing states (FFLO). The ODLRO order parameter, Δ_{op} is defined by the long distance behaviour of the correlation $\langle c^\dagger_{i\uparrow}c^\dagger_{i\downarrow}c_{j\downarrow}c_{j\uparrow}\rangle \rightarrow \Delta^2_{OP}/|U|^2$ for $|\mathbf{r}_i - \mathbf{r}_j| \rightarrow \infty$[21]. The other correlation functions which are of physical significance are the pair-pair ($C_{ij} = \langle c^\dagger_{i\uparrow}c^\dagger_{i\downarrow}c^\dagger_{j\uparrow}c^\dagger_{j\downarrow}\rangle$), density-density ($K^{\sigma\sigma'}_{ij} = \langle c^\dagger_{i\sigma}c_{i\sigma}c^\dagger_{j\sigma'}c_{j\sigma'}\rangle$) and paramagnetic current-current correlation functions, $\Lambda^{\alpha\beta}_{ij} = \langle j_\alpha(r_i)j_\beta(r_j)\rangle$ where j_α is the component of the paramagnetic current density operator at site r_i given by $j_\alpha(r_i) = it\sum_\sigma\left[c^\dagger_{i+\alpha,\sigma}c_{i,\sigma} - c^\dagger_{i,\sigma}c_{i+\alpha,\sigma}\right]$. Here r_i and r_j denote lattice sites. In our numerical computation, a pair (say for C_{ij}) is located at a fixed site r_i and the probability of another pair at site r_j, where r_j can be any other site of the lattice, is calculated. Similar notation is followed for $K^{\sigma\sigma'}_{ij}$ and $\Lambda^{\alpha\beta}_{ij}$ as well. It is useful to mention here that while we have considered site r_j to be along any arbitrary direction with respect to site r_i, however results (shown in Fig. 6-8) are presented for r_j to be along the length (x-axis) of the lattice. The reason is the following. While the direction along which r_j is chosen is immaterial for the homogeneous BCS phase, for the FFLO phase, the preferred direction (for the case of without trap) is set by the propagation of the modulating gap parameter, which is along the length (x-axis) of the lattice (Fig. 2(c)). To retain uniformity, we follow the same for the trapped case, where modulation exists along a ring away from the trap center.

Finally, we comment on the choice of parameters. We perform our studies for an inter-particle attraction strength $|U| = 2.5t$ which may be considered to be weak, and hence suitable as we propose a BCS superfluid as a starting point. It should be noted that further lower values of $|U|$ yield a vanishing gap parameter for any reasonable value of density. We have chosen μ to be $-0.5t$. All quantities are calculated at zero temperature, where the Fermi function has been taken as unity. We have also considered other values for density, ranging from moderate to large (very small densities do not yield a stable superfluid phase) and different (weak) interaction strengths, U, however we have not noted any qualitative change in behaviour of the correlation functions which are on focus in this paper. Further, it may be noted that the gap parameter undergoes a one dimensional modulation[22] with a period that is commensurate with the lattice size in the absence of confining potential. Thus a two dimensional lattice of size 32×16 has been considered so that more periods of modulation of the gap parameter can be accommodated. We have also checked that the results for a 32×32 lattice are unaltered with regard to the modulation of the gap parameter and qualitative behaviour of the correlation functions. In case of presence of the external harmonic potential, the dimension of the lattice is chosen to be 24×24 with open boundary conditions and the trap center is located at the center of the lattice.

III. RESULTS

To characterize the spin polarized phase in presence of a harmonic trap, we have computed various correlation functions in the real space for various values of population imbalance and trap depth. The different correlations that we consider in this work are gap parameter, magnetization, off-diagonal long range order (ODLRO), pair, density and current correlations. The list is not exhaustive but will serve our purpose in providing insights on the nature of the population imbalanced superfluid in the superposed environment of a harmonic trap.

Before we proceed with the discussion on the results for various correlation functions, we wish to distinguish between the three phases, *viz.* the homogeneous BCS phase for low values of magnetic field (*i.e.* low population imbalance), a spatially inhomogeneous FFLO phase for moderate field strengths and finally a *normal* phase at higher magnetic fields. Thus to fix our input parameters, we present a schematic diagram of the BCS and FFLO phases, both in the absence and presence of trap in Fig. 1. The FFLO phase is marked by a lower and a upper critical magnetic field values, h_{c_1} and h_{c_2} respectively and lie intermediate to the BCS and normal phases. These boundaries are obtained from the behaviour of the gap parameter obtained solving Eq. (5). Fig. 1 shows that the values for h_{c_1} and h_{c_2} are $0.35t$ and $0.5t$ in the absence of trap whereas it shifts to $h_{c_1} = 0.25t$ and $h_{c_2} = 0.4t$ when the trapping potential is switched on for $|U| = 2.5t$ and $\mu = -0.5t$. Hence all our results are presented for two values of magnetic field, h which denote two different values of population imbalance. One being in the homogeneous BCS phase ($h < h_{c_1}$) and another in the FFLO phase ($h_{c_1} < h < h_{c_2}$). From Fig. 1, we choose $h = 0.1t$ and $h = 0.5t$ corresponding to no trap and $h = 0$ and $h = 0.3t$ in presence of the harmonic trap for BCS and FFLO phases respectively. Moreover, we have restricted the demonstration and discussion of the results to one value of strength of the trapping potential

viz. $V_0 = 0.016t$. The representative value considered here, approximately denotes a point where there possibly exists a very shallow minimum in the spectral gap. We have investigated the correlations for other values of trapping depths in both the superfluid (small V_0) and strongly confined regimes ($V_0 \sim 0.2t$, see Figs. 5 and 9 and related discussion).

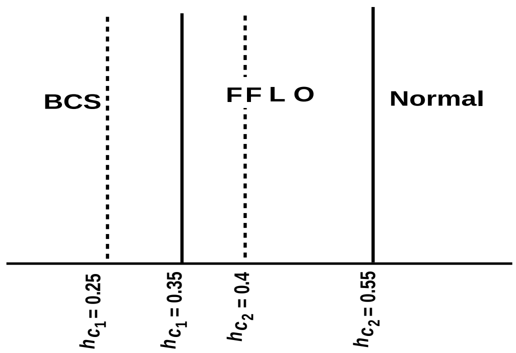

FIG. 1: A schematic representation of the FFLO phase is shown as a function of the magnetic field both in the absence and presence of trap for $|U| = 2.5t$ and $\mu = -0.5t$ (same for all plots). This phase is intermediate to a BCS phase (homogeneous Δ_i) and a normal phase ($\Delta_i = 0$). The boundaries of the FFLO phase are marked by h_{c_1} and h_{c_2}. The solid lines are the boundaries in the absence of trap whereas the broken lines mark the boundary in the presence of the trapping potential of strength, $V_0 = 0.016t$. Same numerical value of V_0 is chosen for Figs. 2-4 and Figs. 6-8. Also all the quantities are in units of hopping integral, t.

We now present our results for the gap parameter, Δ_i in presence of a harmonic confinement and compare with the corresponding behaviour in absence of the trap. It may be noted that Δ_i is homogeneous for low values of magnetic fields (population imbalance being small) in the absence of trap, whereas the trapping effects lead to a maximum of Δ_i at the trap center, hence decreases and finally vanishes away from the center (see Fig. 2 (a) and (b)). Such contrasting behaviour is caused by the trapping effects of the harmonic potential that results in the accumulation of atomic densities around the center thereby leading to such an inhomogeneous distribution of Δ_i in real space. Next we compare Δ_i for the modulating (FFLO) phase. Δ_i modulates along one direction (x-axis) of the lattice without the trap while it modulates along the radial direction in the presence of trap, as shown in Fig. 2 (c) and (d). That is, far away from the trap center, the gap parameter undergoes a sign change confirming the presence of the FFLO phase.

We proceed to discuss results on the local magnetization, m_i ($= \langle n_{i\uparrow}\rangle - \langle n_{i\downarrow}\rangle$) presented in Fig. 3. While m_i for the BCS phase, is zero understandably (being nonmagnetic due to lack of unpaired particles) across the lattice (see Fig. 3(a) and (b)), it modulates with the period half of that of Δ_i in the FFLO phase when trapping effects are not included (Fig. 3(c)). The large values of magnetization occurs at nodal lines with broken pairs, whereas lattice sites with finite Δ_i correspond to weak magnetization, thereby leading to a phase difference between these two quantities. The magnetization profile in the presence of trap, exhibits a bimodal structure (as shown in Fig. 3(d)) since the magnetization is non-zero around the ring like nodal line. In other words, a small number of particles are squeezed into the inner core of the harmonic trap due to pair formation in the superfluid phase, while the unpaired (majority) carriers are pushed out to outside of the core.

We now plot the ODLRO order parameter, Δ_{op} as a function of magnetic field without the harmonic trapping in Fig. 4. It shows sharp drop of Δ_{op} at the onset of the FFLO phase at $h_{c_1} = 0.35t$, thereby indicating weakened superconducting correlations in the FFLO phase (see Fig. 1). Δ_{op} is also computed in the presence of trap but it is seen at two fixed values of magnetic field *viz.* $h = 0$ and $h = 0.3t$ for various values of trapping potential, V_0 (shown in Fig. 5). Our results clearly indicate a higher value of Δ_{op} for $h = 0$ (no population imbalance) than that for $h = 0.3t$, but is suggestive of a rapid decay for both with increasing confinement of the carriers.

Next, we compute pair-pair (C_{ij}), density-density ($K_{ij}^{\sigma\sigma'}$) and current-current ($\Lambda_{ij}^{\alpha\beta}$) correlation functions, the utility of which in experiments are illustrated in literature[23–25]. All these quantities point towards a spatially homogeneous (translationally invariant) ground state for the BCS phase, while the translational invariance is broken in the FFLO phase resulting in spatial inhomogeneities of these correlation functions. The correlation functions mentioned above need individual attention.

We begin with the pair-pair correlation function, C_{ij} (defined in the previous section), the real space scan of which taken along the length of the lattice (along the direction in which Δ_i modulates), from one end to another is presented in Fig. 6(a) and (b). C_{ij} in the BCS phase ($h = 0.1t$) and without an external confinement, is higher (approximately ten times) than the value in the FFLO phase ($h = 0.5t$). This supports weakening of the superconducting correlations with increasing magnetic field as also is evident from Figs. 2(a) and (c). The behaviour of C_{ij} in the presence of trap are interesting in the following sense. It shows a maximum value at trap center and falls off in a near symmetric fashion away from the trap center. It also indicates towards higher correlations in BCS ($h = 0$) which are considerably reduced (approximately to one-third) in the FFLO phase ($h = 0.3t$).

The reason behind C_{ij} showing a hump-like behaviour is as follows. The probability of finding a pair in the center of the trap is higher as the underlying trapping potential is minimum (deep) at the center. However, it drops at the edges where the trapping potential is high (shallow). Additionally, C_{ij} drops significantly as the external trapping potential is switched on, suggesting the fact that the trapping potential assists an imbalanced superfluid phase to have weakened pair-pair correlations.

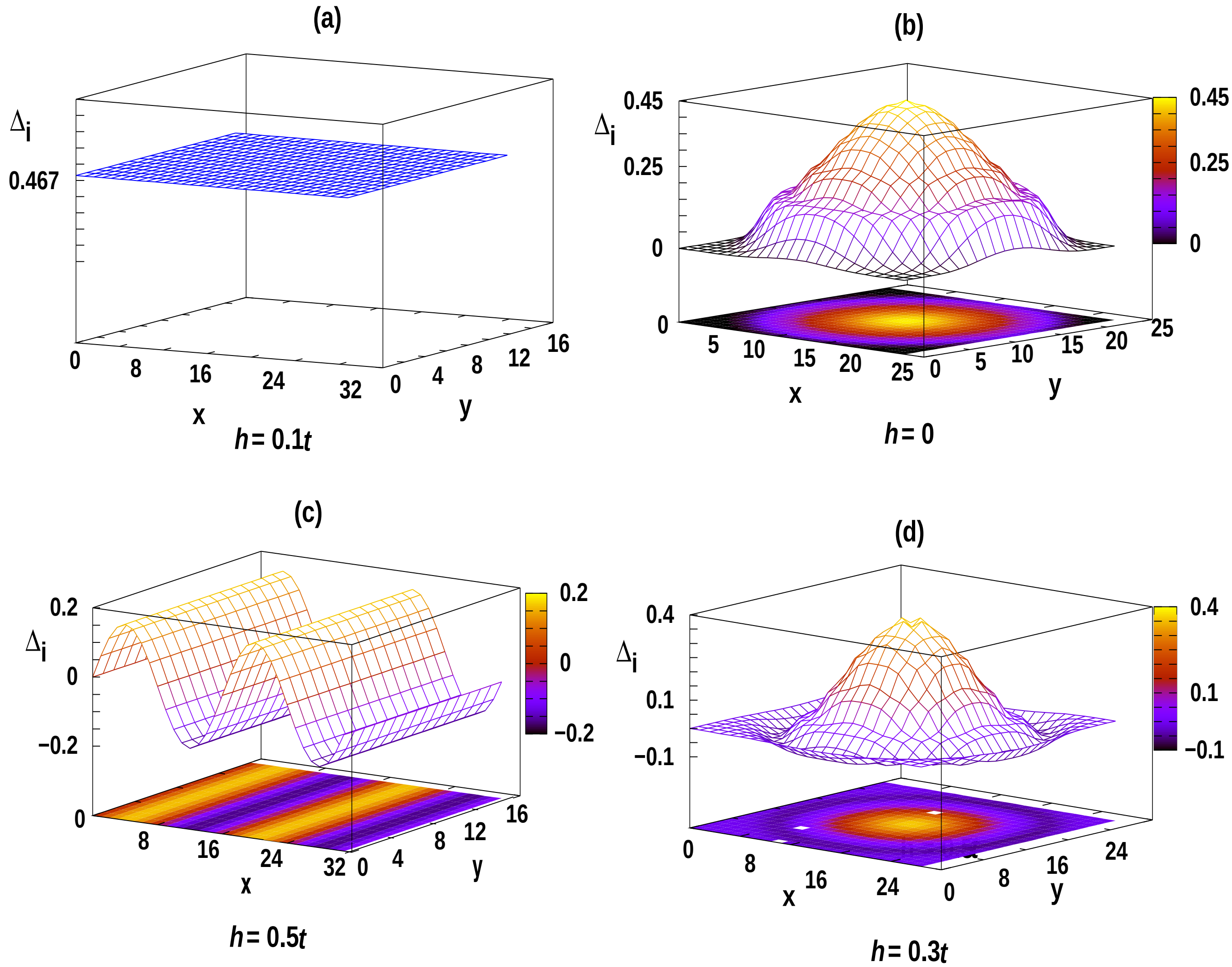

FIG. 2: (Colour online) Local pairing amplitude, Δ_i in BCS ((a) and (b)) and modulating FFLO ((c) and (d)) phases are shown. Figs. (a) and (c) are for cases without trap while (b) and (d) are in presence of trap. The system size is 32×16 in the absence of trap and 24×24 when the trapping potential is present and are same for all our results.

Hence we discuss density-density correlation function, $K_{ij}^{\sigma\sigma'}$. In absence of the trap, there is no significant difference between the behaviour of $K_{ij}^{\sigma\sigma'}$ in the BCS and FFLO phases (see Fig. 7(a)) with both remaining constant throughout the lattice. However, the correlations show a hump-like feature as the trapping potential is turned on (Fig. 7(b)). Though the shape of the plot is similar to that of C_{ij}, the qualitative behaviour in both the phases is reversed *i.e.* $K_{ij}^{\sigma\sigma'}$ for the FFLO phase ($h = 0.3t$) is higher than the BCS phase ($h = 0$). It may be noted that we have considered all $\sigma, \sigma' =\uparrow, \downarrow$. However the results are only shown for $\sigma = \sigma' =\uparrow$. Other choices yield qualitatively same results. A possible explanation for the rise in correlations between different species ($\uparrow$ and $\downarrow$) of atoms for the FFLO phase can be explained in terms of the increase in number of unpaired particles with the increase in the magnetic field. Thus the correlations are higher in the FFLO phase (with more number of unpaired carriers) as compared to the BCS phase where all carriers are paired. Further, a comparison of $K_{ij}^{\sigma\sigma'}$ in the presence of trap to the one without trap shows remarkable reduction in the amplitude in the presence of trap. These correlations are particularly useful in *time of flight* experiments in which the trapping potential is suddenly switched off, resulting in free expansion of the atomic gas and thus enable realization of the velocity or momentum profile[24]. These techniques have been used to detect the nature of the condensate *e.g.* the superfluidity of the atoms constituting the condensate etc[26].

Finally, we analyze the behaviour of the paramagnetic current-current correlations (with $\alpha = \beta = x$), *i.e.* Λ_{ij}^{xx} in an environment of harmonic confinement. Λ_{ij}^{xx} rapidly decays to zero in the BCS phase, while it modulates across the lattice and remains finite in the FFLO phase, irrespective of the value of the trapping potential

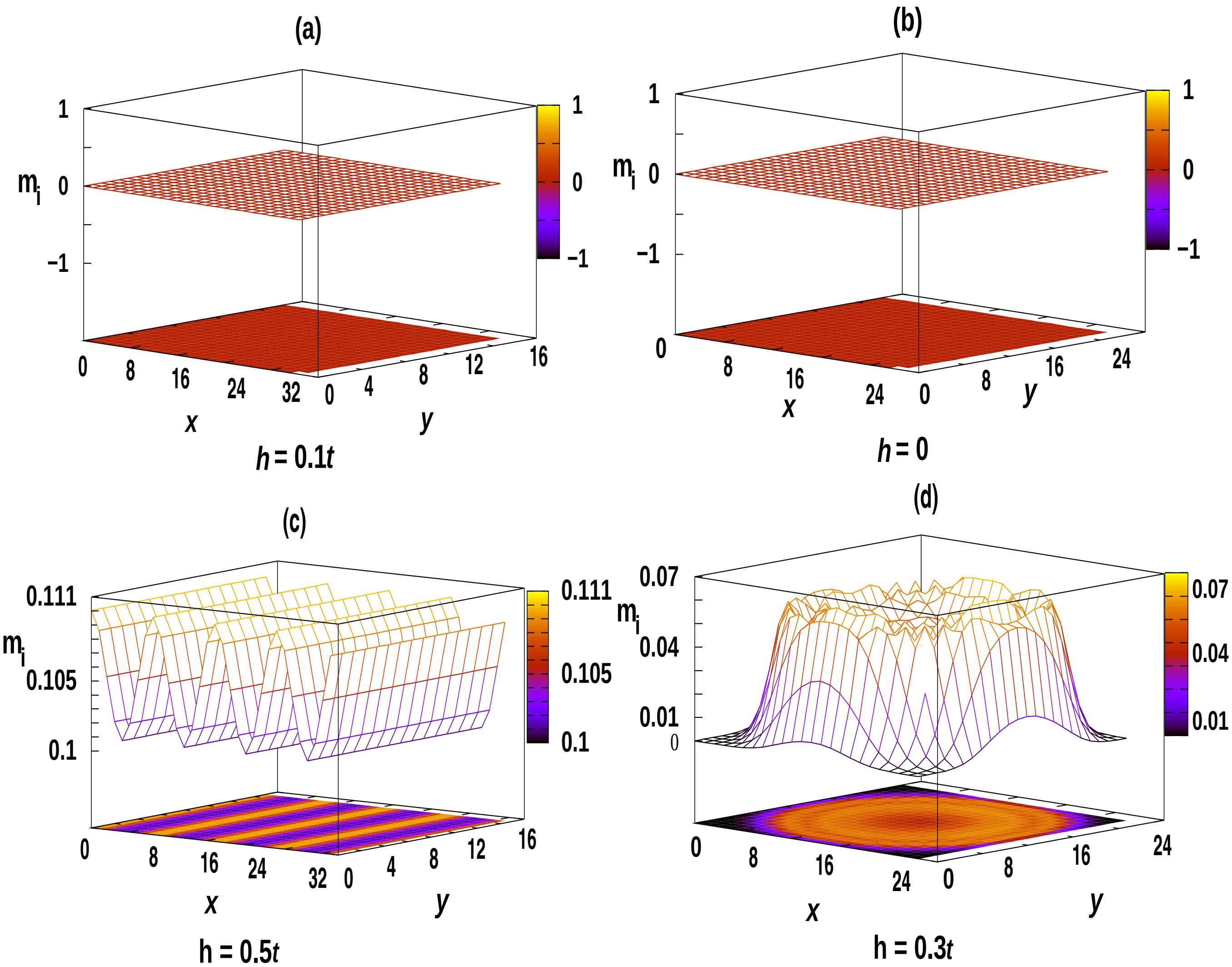

FIG. 3: (Colour online) Local magnetization, m_i is shown in BCS (a) and (b) and FFLO (c) and (d) phases both in the absence and presence of the confining potential.

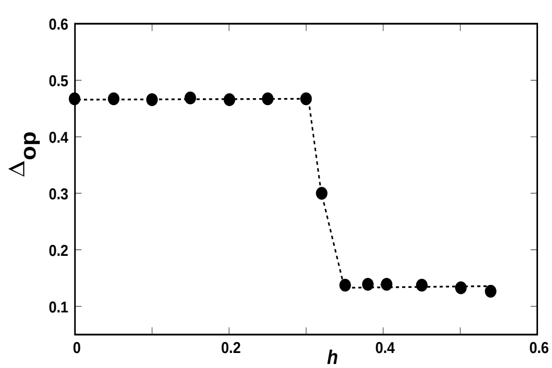

FIG. 4: The ODLRO order parameter, Δ_{OP} (in units of t) is shown as a function of magnetic field, h. The dotted line is a guide to the eye and used in some of the subsequent plots.

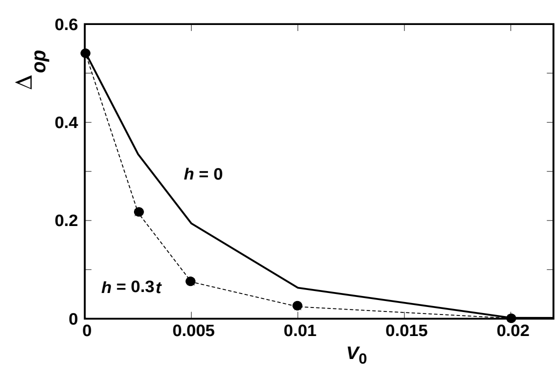

FIG. 5: The ODLRO order parameter, Δ_{OP} (in units of t) is shown as a function of trapping potential, V_0 for $h = 0$ (bold line) and $h = 0.3t$ (points and dashed line).

as shown in Fig. 8(a) and (b). The modulation in real space does not have a uniform profile as Δ_i (Fig. 2(c)), possibly because of the involvement of more than one momentum vector in Λ_{ij}^{xx}. Nevertheless, we still have the

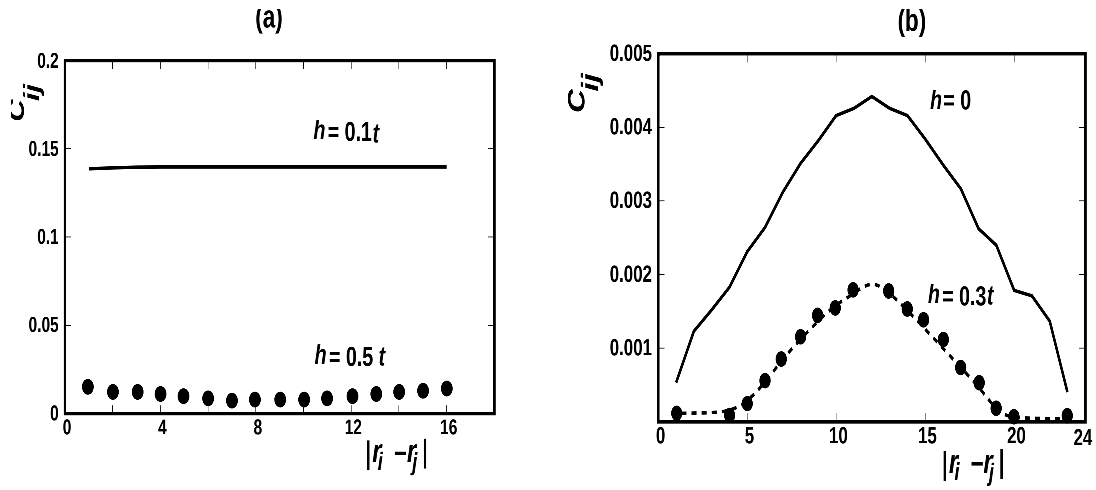

FIG. 6: Pair-pair correlation function, C_{ij} is shown across the lattice in BCS and FFLO phases without trap (a) and compared with the case when the trapping effects are invoked (b).

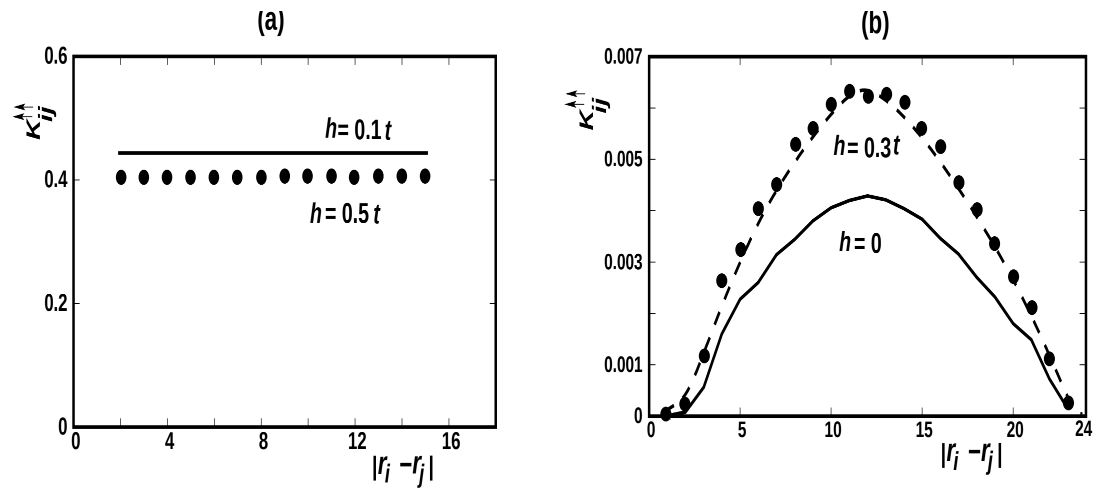

FIG. 7: Density-density correlation function for $\sigma = \sigma^{'} = \uparrow$ *i.e.* $K_{ij}^{\uparrow\uparrow}$ is shown across the lattice in BCS and FFLO phases without trap (a) and with trap (b).

response of the system to an external perturbation, to be more in the FFLO phase than that in the BCS phase and thus in turn reiterates the presence of stronger superconducting correlations in the homogeneous BCS phase. We have recorded higher values for Λ_{ij}^{xx} for the case where no trap effects are included than those in presence of the trap. The likely reason must be the reduced mobility of the charge carriers, thereby causing weakened current vectors in presence of the trap due to localization effects.

In the following discussion, we describe the scenario corresponding to strong confining effects. As the trap is made deeper, in order to keep the chemical potential fixed (needed to obtain a population imbalanced state and FFLO phase to be realized), the particle density is lowered. So the system looses particles and heads towards an *empty* state with only a few particles remaining at the core of the potential as trapping effects are made stronger (table I).

In order to obtain a detailed understanding of what happens at larger trap depths, we compute certain physical properties that characterize the state. The gap parameter (Δ_i) vanishes for this scantily populated system (not shown here). The spectral gap, E_{gap} which is defined as the difference in energy between the ground state and lowest lying excited state of the BdG spectrum as obtained by numerically solving Eq. (4), shows an increase with increasing V_0. There is possibly a very shallow minimum that exists around $V_0 \sim 0.016t$ (Fig. 9(a)) as seen in our numerical computation, however this feature is not clear from Fig. 9(a). Since an increasing E_{gap} suggests the onset of an insulating behaviour[27], we explored the average kinetic energy of the particles (Fig. 9(b)) as a measure of their mobilities and find a rapid decay as V_0 is increased.

All these results put together may suggest of the onset of an insulating phase at large trap depths, however when

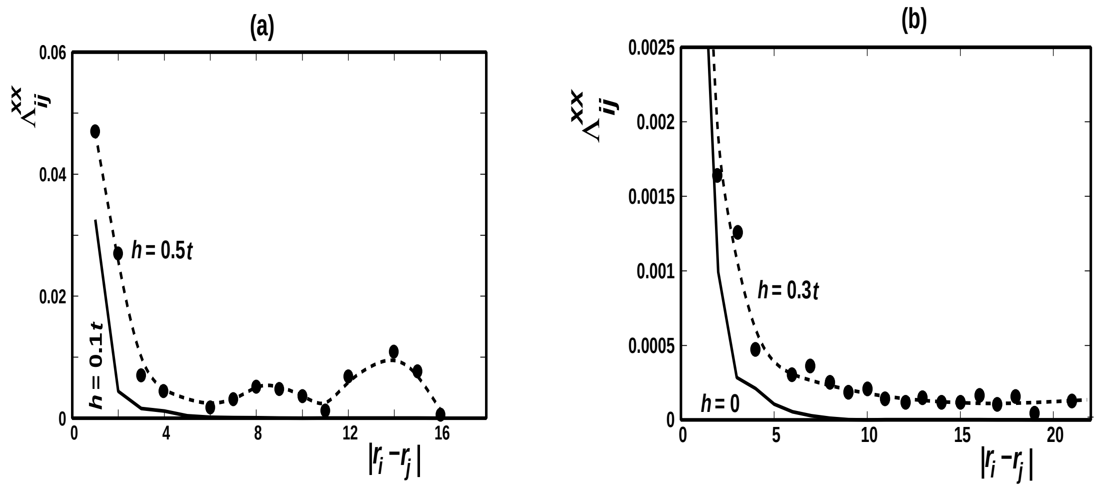

FIG. 8: Current-current correlation for $\alpha = \beta = x$ *i.e.* Λ^{xx}_{ij} is shown across the lattice in BCS and FFLO phases without trap (a) and with trap (b).

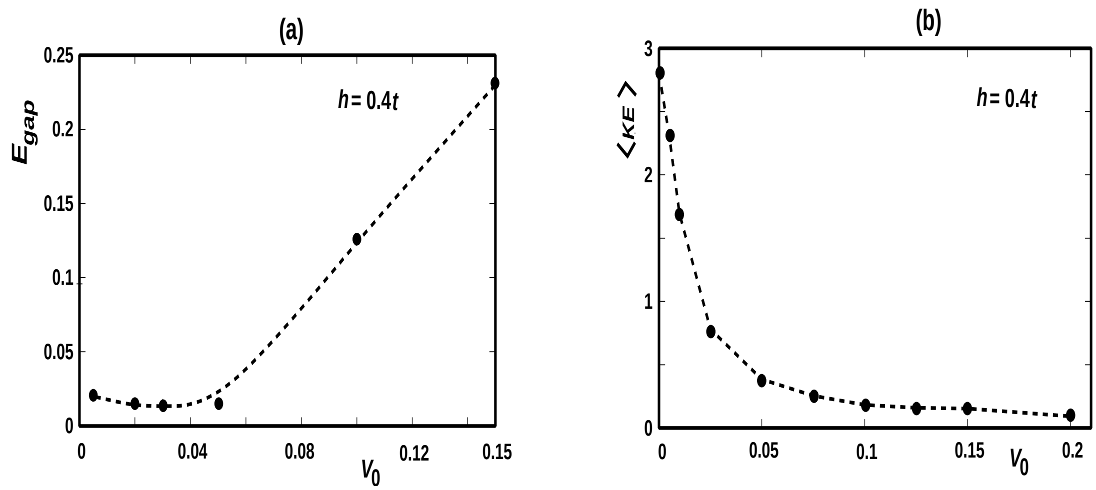

FIG. 9: (a) The spectral gap E_{gap} is shown as a function of increasing trapping potential V_0. (b) The average kinetic energy of the charge carriers, $\langle KE \rangle$ is plotted as a function of V_0. Both plots are presented at $h = 0.4t$.

taken together with the fact that particles are gradually *leaking out* (see table I), they appear to be artifacts of emptying of the system.

IV. CONCLUSIONS

In summary, we have explored the ground state of a population imbalanced Fermi gas on a two dimensional lattice in presence of a harmonic confinement. We adopted a weak coupling s-wave superconductor described by an attractive Hubbard model in presence of imbalanced spin species induced by a Zeeman field. The ground state is inhomogeneous for a range of magnetic field strengths which is characterized by a modulating order parameter, *i.e.* the FFLO phase. The exotic FFLO phase is obtained both in presence and absence of the trap.

To aid our understanding of the trapping effects on the system, we computed various correlation functions *e.g.* off-diagonal long range order, pair-pair, density-density and current-current correlations in real space and provided adequate comparisons with the scenario when the confining potential is absent. The results are indicative of weakened correlations as population imbalance is increased and confining effects are invoked.

Larger confining effects induce loss of particles from the system (chemical potential being fixed to its original value). At large values of V_0, the system becomes almost *empty* of particles. The physical properties of this *empty* system behaves similar to that of an insulator. However such signatures should *not* be considered indicative of a transition from a superfluid to an insulating phase as they are artifacts of a near *empty* system.

TABLE I: The total density of particles, $\langle n \rangle (= \sum_{i,\sigma} \langle n_{i\sigma} \rangle)$ is shown as a function of increasing trapping strength, V_0 (in units of t). The density drops significantly as the trapping strength is made stronger leading to an *empty* system at large values of V_0.

Trapping strength (V_0)	Total density ($\langle n \rangle$)
0.00	0.6361
0.016	0.2135
0.05	0.0659
0.10	0.0313
0.20	0.0156

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